ANTENNA MUTUAL COUPLING EFFECTS ON CORRELATION, EFFICIENCY AND SHANNON CAPACITY

Anders Stjernman

Antenna Research Center, Ericsson Research, Ericsson AB, SE 417 56 Göteborg, Sweden

Email: anders.stjernman@ericsson.com

ABSTRACT

MIMO (multiple-input-multiple-output) systems make use of multiple antennas at both ends of a communication link to exploit the spatial dimension for increasing the capacity. Correlation of the antenna signals has an affect on the capacity of a MIMO implementation. High correlation levels occur mainly for compact antenna realizations, where the separation of the antenna elements is small and the mutual coupling is strong. The antenna mutual coupling distorts the antenna radiation patterns and the input impedances, and therefore affects the correlation level and the efficiency. The correlation level is decreased by coupling, which would improve the Shannon capacity, but the effiency of the antenna system is also decreased, which leaves the Shannon capacity nearly unchanged as compared to the case when neglecting coupling effects. The Shannon capacity is increased in a interference limited receiving situation and can also be increased in a transmitting situation in a narrow frequency band by using a decoupling network.

1. INTRODUCTION

The concept of total Shannon capacity for a MIMO antenna system is introduced. Typically only the Shannon capacity at a single frequency is discussed, assuming that the system is narrow band and the properties of the antenna system is invariant over the frequency band. For an OFDM (orthogonal frequency division multiplexing) system with many narrow frequency bands, those assumptions could be valid for each band. However for densely packed antenna elements, the properties could vary rapidly between the different used frequency bands. A differential Shannon capacity could be introduced for each used frequency band and the total Shannon capacity as the sum of those differential Shannon capacities.

The coupling has mainly two effects; a decrease of antenna efficiency and thereby reduced (SNR) signal-to-noise-ratio and Shannon capacity, and a reduction of correlation between received signals increasing the Shannon capacity. Usually, only the later effect is considered, leading to the conclusion that coupling is beneficial for a MIMO system. However, in reality the decrease in SNR is more severe than the correlation effect, at least in a noise limited environment.

The coupling between the antenna elements could be counteracted by using passive coupling compensation networks; however complete coupling cancellation and matching can only be achieved for a single frequency.

In this paper, typical total Shannon capacities are shown as a function of separation between two half wavelength long dipoles. The theory could be extended to arbitrary number of antenna elements.

2. SHANNON CAPACITY

The Shannon capacity of a communication system is the theoretical upper limit for the capacity that can be reached. One distinguishes between informed and uninformed transmitter. In the case of informed transmitter water filling is used and the Shannon capacity is given by [1]

$$C = \sum_{N'} \log_2(1 + \lambda_i P_i) = \sum_{N'} \log_2(\lambda_i D) \qquad (1)$$

where $1/\lambda_i + P_i = D, i = 1 \dots N'$ and $\sum_{N'} P_i = P$. The total available SINR (signal to interference and noise ratio) is P and λ_i are the eigenvalues of the Gram matrix

$$\mathbf{G} = \mathbf{H}^{\mathrm{H}}\mathbf{H} \qquad (2)$$

where $\mathbf{H}$ is the channel transfer matrix, including antennas and the propagation environment surrounding the antennas. $\mathbf{H}^{\mathrm{H}}$ is the Hermtian of $\mathbf{H}$. N' is chosen so that all $P_i > 0$, i.e. D is calculated from $D = (P + \sum_{N'} 1/\lambda_i)/N'$, starting with $N' = N$, the number of eigenvalues, and if the resulting $D < 1/\lambda_{N'}$, N' is reduced by 1. It is assumed that the eigenvalues are ordered such that $\lambda_i \geq \lambda_{i+1}$ for all i.

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In the case of uninformed transmitter the Shannon capacity is given by

$$C = \sum_{N} \log_2(1 + \lambda_i P/N) \tag{3}$$

where the power is equally split between the transmit antennas.

In common practice, the Gram matrix $\mathbf{G}$ is normalized in such way that the diagonal elements are all equal to 1, and hence $\sum_N \lambda_i = N$, but this is not appropriate, when comparing different antenna systems in a given environment. In this case, losses in the antenna systems have to be considered.

3. ANTENNAS IN A RAYLEIGH FADING ENVIRONMENT

The channel matrix $\mathbf{H}$ is dependent on both the antenna properties and the scattering environment surrounding the antennas. A specially interesting environment is the 3D flat Rayleigh fading environment [2], where the rays incident on the receiving antennas have random directions and polarizations. This environment can be seen as an average environment when the orientation of the antenna system is unknown, which often is the case for mobile terminals.

Assuming that the investigated antennas are used as transmitting antennas, and that the corresponding receiving antennas in the 2 by 2 MIMO system, are placed sufficiently far from each other so there is no additional correlation between the received signals, the expectation value of the elements $G_{i,k}$ in the Gram matrix $\mathbf{G}$ can then be expressed as an integral

$$G_{i,k} = \frac{1}{4\pi} \oint \vec{F}_i \cdot \vec{F}_k^* d\Omega \tag{4}$$

between the gain normalized far field patterns $\vec{F}_i$ of the antennas. The gain normalization takes the effiency of the anntenna system into account.

Neglecting coupling between the antennas, and using the undisturbed patterns, the integral in Eq. 4 evaluates as

$$G_{i,k} = \Re(Z_{i,k})/\sqrt{\Re(Z_{i,i})\Re(Z_{k,k})}, \tag{5}$$

where $\Re(Z_{i,k}) = R_{i,k}$ are the real part of the elements of the admittance matrix $\mathbf{Z}$.

Taking into account the coupling effects, the following relationship is valid if one assumes lossless antennas [3]:

$$\mathbf{G} = \mathbf{I} - \mathbf{S}^{\mathrm{H}}\mathbf{S} \tag{6}$$

where $\mathbf{I}$ is the unit matrix and $\mathbf{S}$ is the scattering matrix referenced at the antenna ports.

3.1. Two identical parallel dipoles

As an example consider two identical half wavelength long dipoles. They are parallel and the are separated with the distance d from each other. Assuming that the dipoles are minimum scattering antennas the normalized overlap integral between the far field patterns defined as

$$\rho = \frac{\oint \vec{F}_1 \cdot \vec{F}_2^* d\Omega}{\sqrt{\oint \vec{F}_1 \cdot \vec{F}_1^* d\Omega \oint \vec{F}_2 \cdot \vec{F}_2^* d\Omega}} \tag{7}$$

evaluates as

$$\rho = \frac{R_{12}}{R_{11}}$$

when neglecting the coupling and as

$$\rho = \frac{R_{12}}{R_{11}} \frac{R_{12}^2 + X_{12}^2}{4R_{11}^2 - 3R_{12}^2 + X_{12}^2}$$

when the coupling is considered and the antennas are individually matched to free space, i.e. they are each loaded with the complex impedance $R_{11} - jX_{11}$. The overlap integral is identical to the correlation between the received signals in the considered environment and the impedance matrix is

$$\mathbf{Z} = \begin{bmatrix} R_{11} + jX_{11} & R_{12} + jX_{12} \\ R_{12} + jX_{12} & R_{11} + jX_{11} \end{bmatrix}.$$

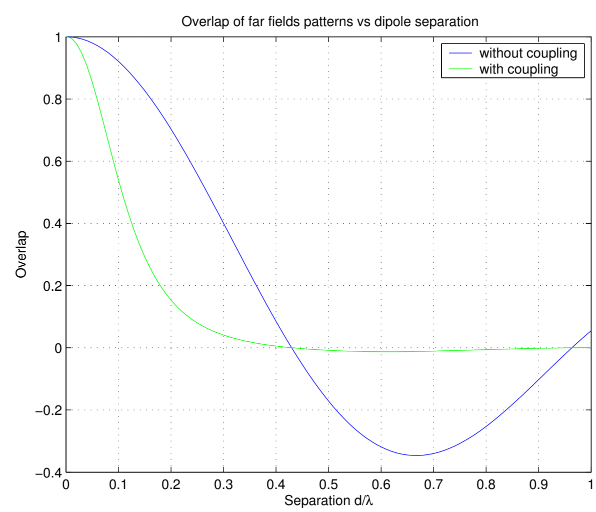

Figure 1. The overlap integral between the far fields of two parallel half wavelength long dipoles vs separation

A comparison between the two expressions versus the dipole separation is shown in Fig. 1. It is clearly shown that the overlap is dramatically reduced when the coupling is considered.

However, the coupling is not only responsible for the overlap between the patterns, it also effects the radiated power of the antenna system, since some of the transmitted power ($|S_{11}|^2$) is reflected back to the transmitter and some ($|S_{12}|^2$) is scattered to the other transmitter. The radiated power is thus reduced by a factor $1 - |S_{11}|^2 - |S_{12}|^2$, and by assuming the same receiving situation as for one antenna, this factor has to be accounted for.

The eigenvalues of the Gram matrix from Eq. 6 is in the case of two identical lossless antennas given by

$$\lambda_1 = 1 - |S_{11} - S_{12}|^2 \quad (8)$$
$$\lambda_2 = 1 - |S_{11} + S_{12}|^2 \quad (9)$$

and they are displayed in Fig. 2 together with the eigenvalues one gets when neglecting coupling, namely

$$\lambda_1 = 1 + R_{12}/R_{11} \quad (10)$$
$$\lambda_2 = 1 - R_{12}/R_{11}. \quad (11)$$

It is clearly seen that the coupling compensated eigenvalues never exceed 1, while the largest eigenvalue approaches 2 when neglecting the coupling.

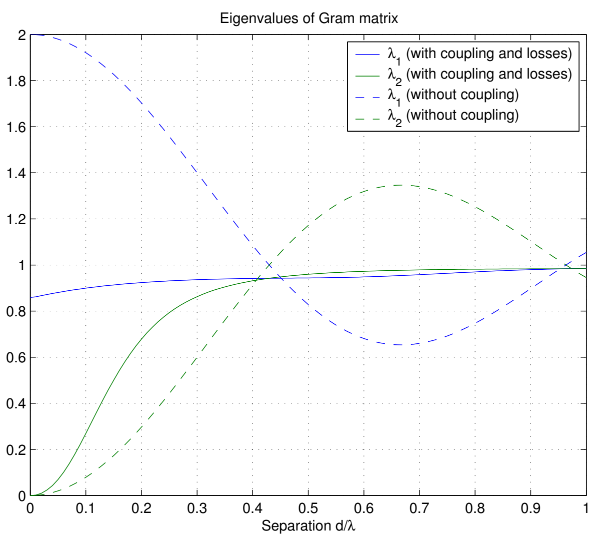

Figure 2. The eigenvalues of the Gram matrix of two parallel half wavelength long dipoles vs separation

Using the eigenvalues displayed in Fig. 2 result in the Shannon capacities displayed in Fig. 3 for some various values of signal to noise ratio. The resulting capacities are surprisingly similar despite the large difference between the eigenvalues for the two different assumptions. Notice however that the Shannon capacity decreases with decreasing separation for SNR=0 dB when taking the coupling into account, while it increases when neglecting the coupling.

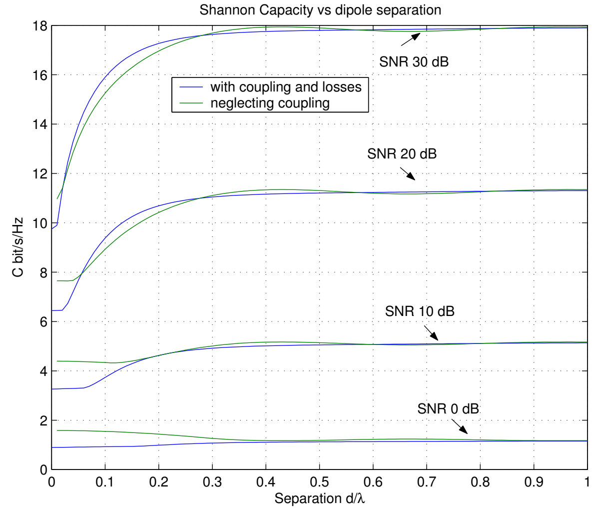

Figure 3. The Shannon capacity of two parallel half wavelength long dipoles vs separation and signal to noise ratio

4. RECEIVING ANTENNAS WITH INTERFERENCE

The same expression for Shannon capacity is valid in the reverse situation, when the investegated antennas are used as receiving antennas instead, and there is no interference. With interference present, the interfering noise is attenuated with the same amount as the signal, counteracting to a certain extent losses in the system.

Using the eigenvector and eigenvalue decomposition of the Gram matrix in Eq. 4 we find that both the signals and the interference is multiplied with the eigenvalues of the Gram matrix, while we could expect the noise level in the receivers being the same as for a single antenna system. The resulting signals and noise at the receiver end applying appropriate weigths are then given by

$$S_i = \lambda_i S_0 I_i = \lambda_i I_0 N_i = N_0 \quad (12)$$

where S_0, I_0 and N_0 are the signal, interference and noise levels for the single antenna case. If the signals incident on the antenna system are uncorrelated, they will still be so after reception by the antenna system. The signal to interference and noise ratio correspoding to the eigenvalue λ_i in the expression Eq. 1 is thus changed to

$$\frac{S_i}{I_i + N_i} = \frac{\lambda_i S_0}{\lambda_i I_0 + N_0} = \lambda_i \frac{1+\alpha}{\lambda_i \alpha + 1} \frac{S_0}{I_0 + N_0} \quad (13)$$

where $\alpha = I_0/N_0$ is the interference to noise ration INR. Hence the eigenvalues λ_i in Eq. 1 should be replaced with $\lambda_i \frac{1+\alpha}{\lambda_i \alpha + 1}$ and thus giving

$$C = \sum_{N'} \log_2(1 + \lambda_i \frac{1+\alpha}{\lambda_i \alpha + 1} P_i) \quad (14)$$

where P_i is calculated with the same algorithm as before but with the eigenvalues replaced accordingly. The effect of interference is displayed in Fig. 4.

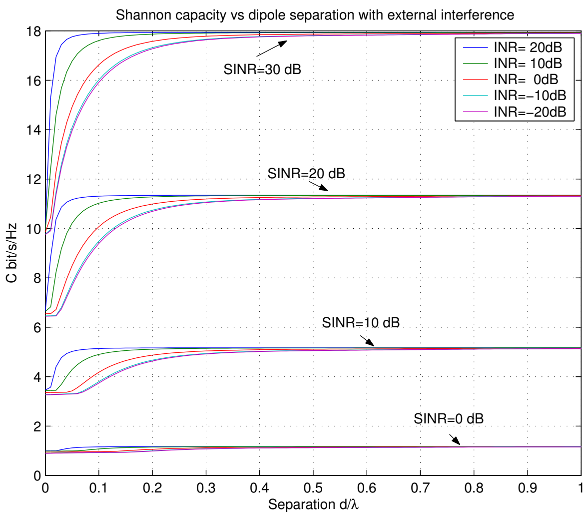

Figure 4. The Shannon capacity of two parallel half wavelength long dipoles vs separation and signal to interference and noise ratio for different interference to noise ratios

Two dipoles can thus be placed with a separation of only 0.03λ if the expected interference to noise ration is 20 dB or more, without losing any MIMO performance.

5. MATCHING WITH COUPLING COMPENSATING ELEMENTS

So far we have only discussed the eigenvalues and Shannon capacity at a single frequency for antennas matched to free space. It is however possible to build a lossless matching network eliminating both S_{11} and S_{12} simultaneously, by introducing decoupling elements between the antennas, leading to both eigenvalues being equal to one. The Shannon capacity would then be $2\log_2(1+P/2)$ independently on the separation between the dipoles, meaning that the dipoles could be placed arbitrarily close to each other, without losing any performance of the system.

The match can however only be achieved at a single frequency f_0, and when the separation is small, the Shannon capacity decreases rapidly with increasing frequency offset as shown in Fig. 5 for 0.01λ separation. Comparing this case with the situation when the dipoles are matched to free space as in Fig. 6 one sees that there is a narrow peak around the center frequency and flat plateau approximately at the flat level in the free space case.

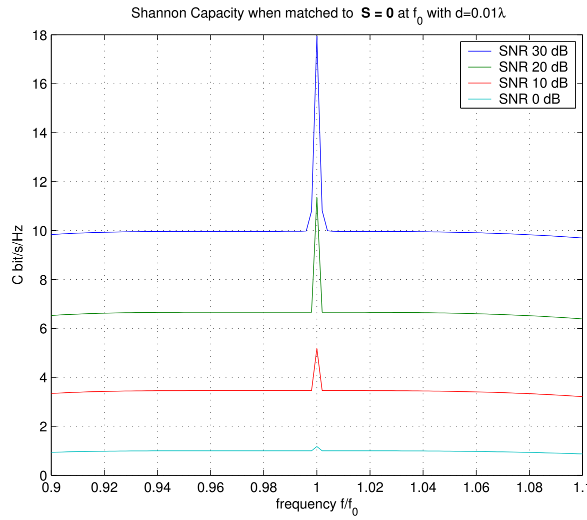

Figure 5. Shannon capacity of two parallel half wavelength long dipoles separated by 0.01λ and matched to $\boldsymbol{S} = \boldsymbol{0}$ at the frequency f_0.

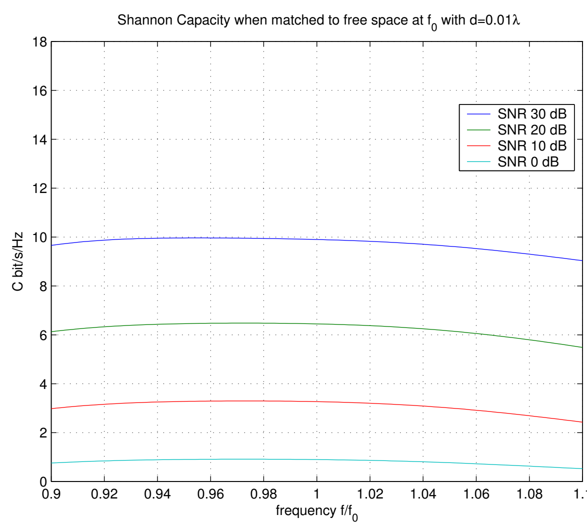

Figure 6. Shannon capacity of two parallel half wavelength long dipoles separated by 0.01λ and matched to free space at the frequency f_0.

When the separation is larger the decrease in capacity for the $\mathbf{S} = \mathbf{0}$ case is slower as shown in Fig. 7 for 0.10λ separation, but on the other hand, the effect of coupling compensation is less pronounced comparing with the free space matched case displayed in Fig. 8.

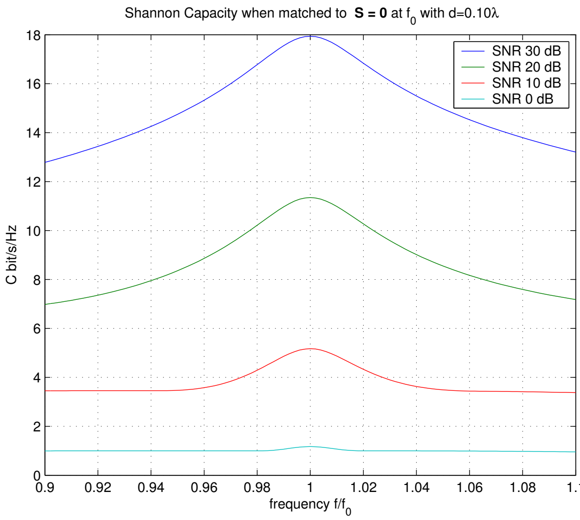

Figure 7. Shannon capacity of two parallel half wavelength long dipoles separated by 0.10λ *and matched to* $\boldsymbol{S} = \boldsymbol{0}$ *at the frequency* f_0.

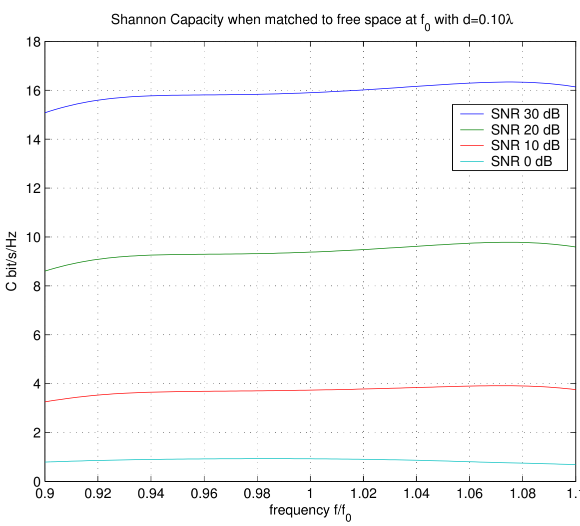

Figure 8. Shannon capacity of two parallel half wavelength long dipoles separated by 0.10λ *and matched to free space at the frequency* f_0.

5.1. Average Shannon Capacity

Assuming that we have an OFDM system with many narrow frequency bands we can benefit from rapid change in Shannon capacity. Taking the average capacity over the total system bandwidth we obtain the values displayed in Fig. 9 for various system bandwidths.

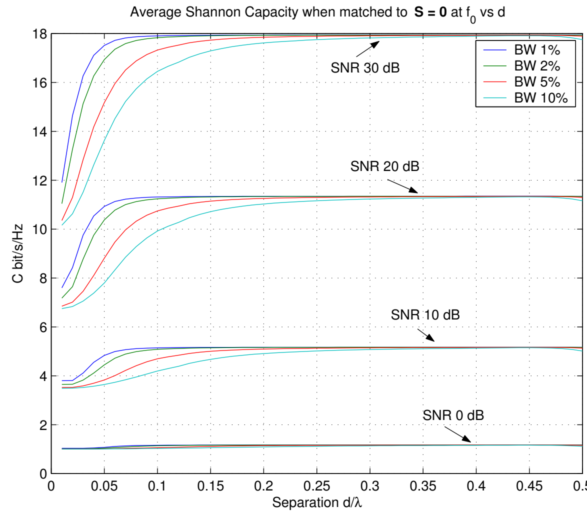

Figure 9. Average Shannon capacity taken over various relative bandwidths of two parallel half wavelength long dipoles and matched to $\boldsymbol{S} = \boldsymbol{0}$ *at the frequency* f_0.

Narrow band systems benefit more from coupling compensation than wide band systems do, and the antennas can be put with rather narrow separation and still render a substantial increase in MIMO capacity compared with the free space case. Since the Shannon capacity does not vary much over frequency when the antennas are matched to free space, the average capacity is the same as the center frequency capacity displayed in Fig. 3 earlier.

6. CONCLUSIONS

It has been shown that the Shannon capacity is not altered significantly, when the coupling is accounted for correctly compared with neglecting coupling, although the correlation or rather the overlap between far field patterns is radically reduced by coupling. This is due to losses in the transmitted or received signals, that counteract the improvement due to decreased correlation. Further it has been shown that the losses, and thereby the coupling can be eliminated by a decoupling network. This network will however be extremely narrow band, when the separation between the antennas is small, however for narrow band systems, an improvement in capacity can be obtained with fairly closely separated antennas (down to 0.03λ for a relative bandwidth of 2 %).

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